On deterministic ACA

Enrico Formenti
University of Nice-Sophia Antipolis,France.

Something that is not synchronous
is asynchronous!

The definition

Something that is not synchronous
is asynchronous!

Motivations

- Concurrent Programming

Motivations

- Concurrent Programming

$$I = \{i_1, i_2, \ldots, i_k\}$$

$$\theta \subseteq I \times I$$

symmetric
irreflexive

$$\theta^c = I \times I \setminus \theta$$

$$\mathcal{G} = \langle I, \theta^c \rangle$$

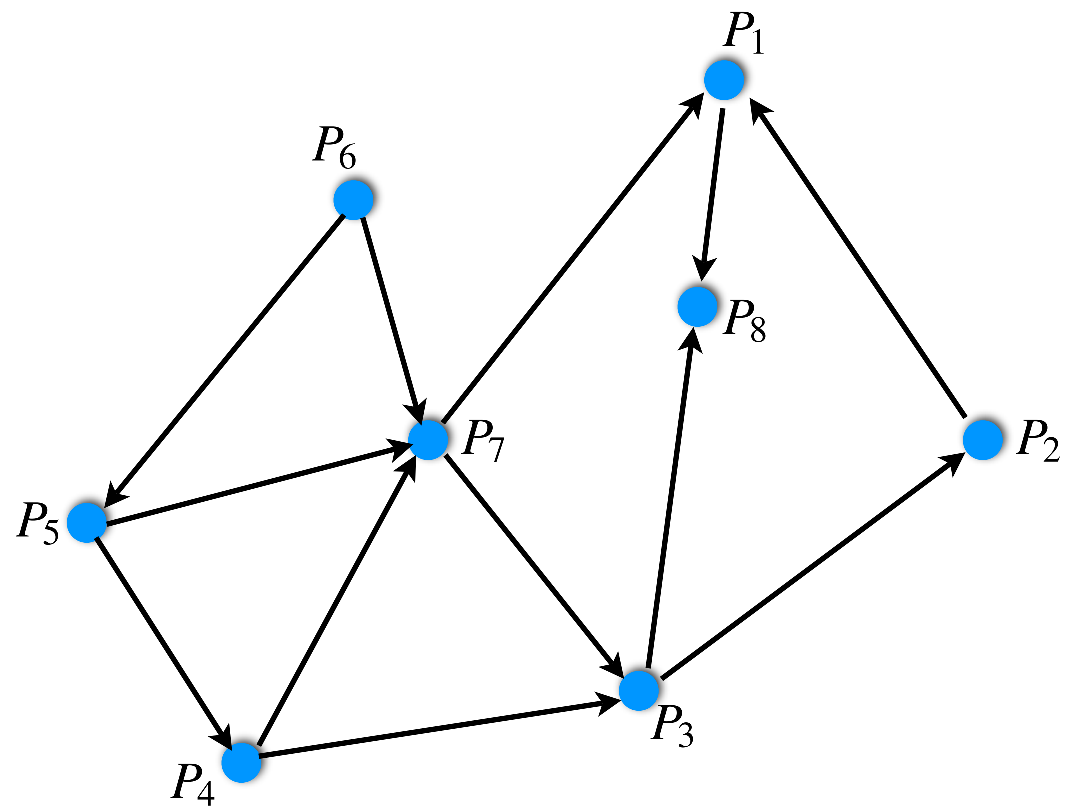
$$\alpha \subseteq I, \theta^c(\alpha) = \{i \in I, \exists j \in \alpha \mid (i,j)\theta^c\}$$

Motivations

- Concurrent Programming

$$P_i \subseteq I \qquad i \in \{1, \ldots, n\}$$

$$G_P$$

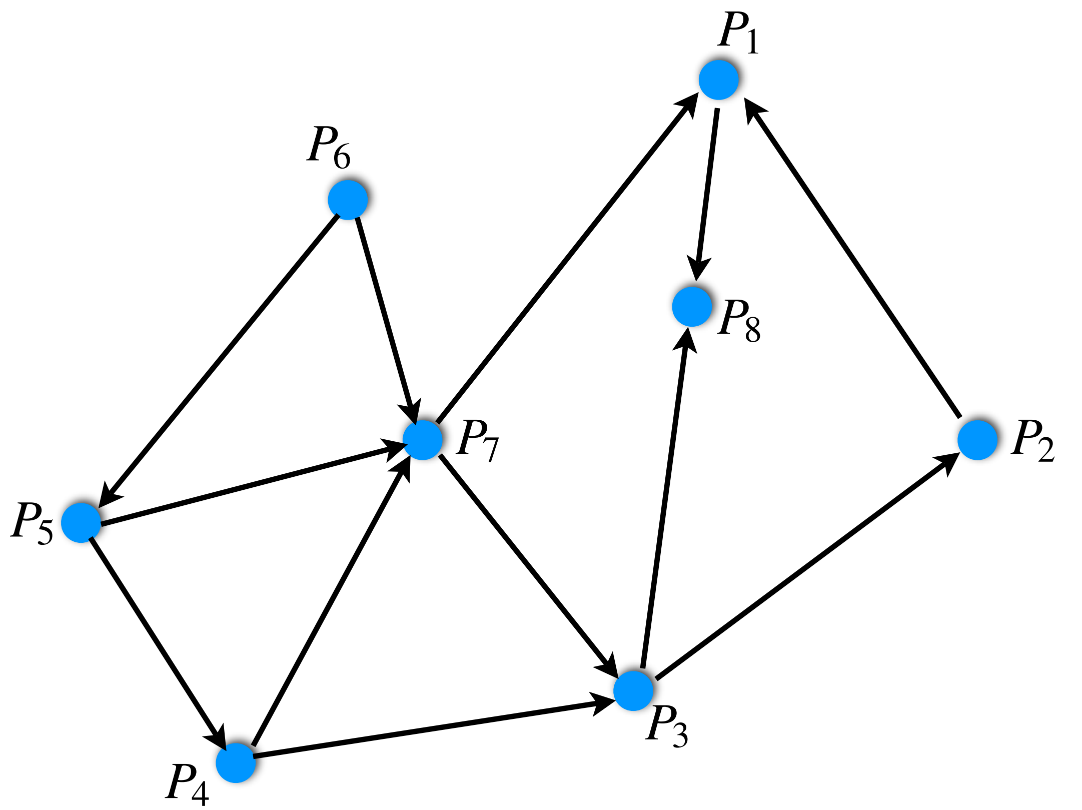


Motivations

- Concurrent Programming
- Bio-informatics

$$P_i \subseteq I \qquad i \in \{1, \ldots, n\}$$

G_P



“Generalizing”

- Finite vs Infinite
- Finite Graph vs Lattice
- Non-uniform vs Uniform
- Drop dependency graph

“Generalizing”

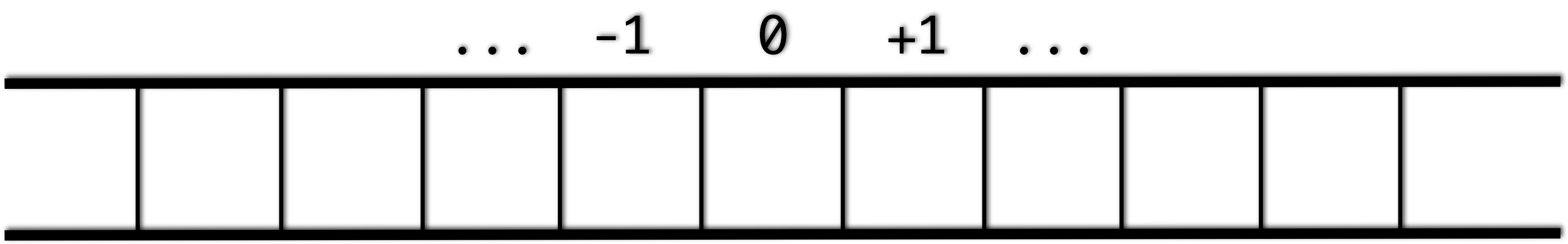
- Finite vs Infinite
- Finite Graph vs Lattice
- Non-uniform vs Uniform
- Drop dependency graph

“Generalizing”

$\mathbb{Z}$ Lattice

$I = \{i_1, i_2, \ldots, i_k\}$ States

$\delta : I^{2r+1} \rightarrow I$ Local rule

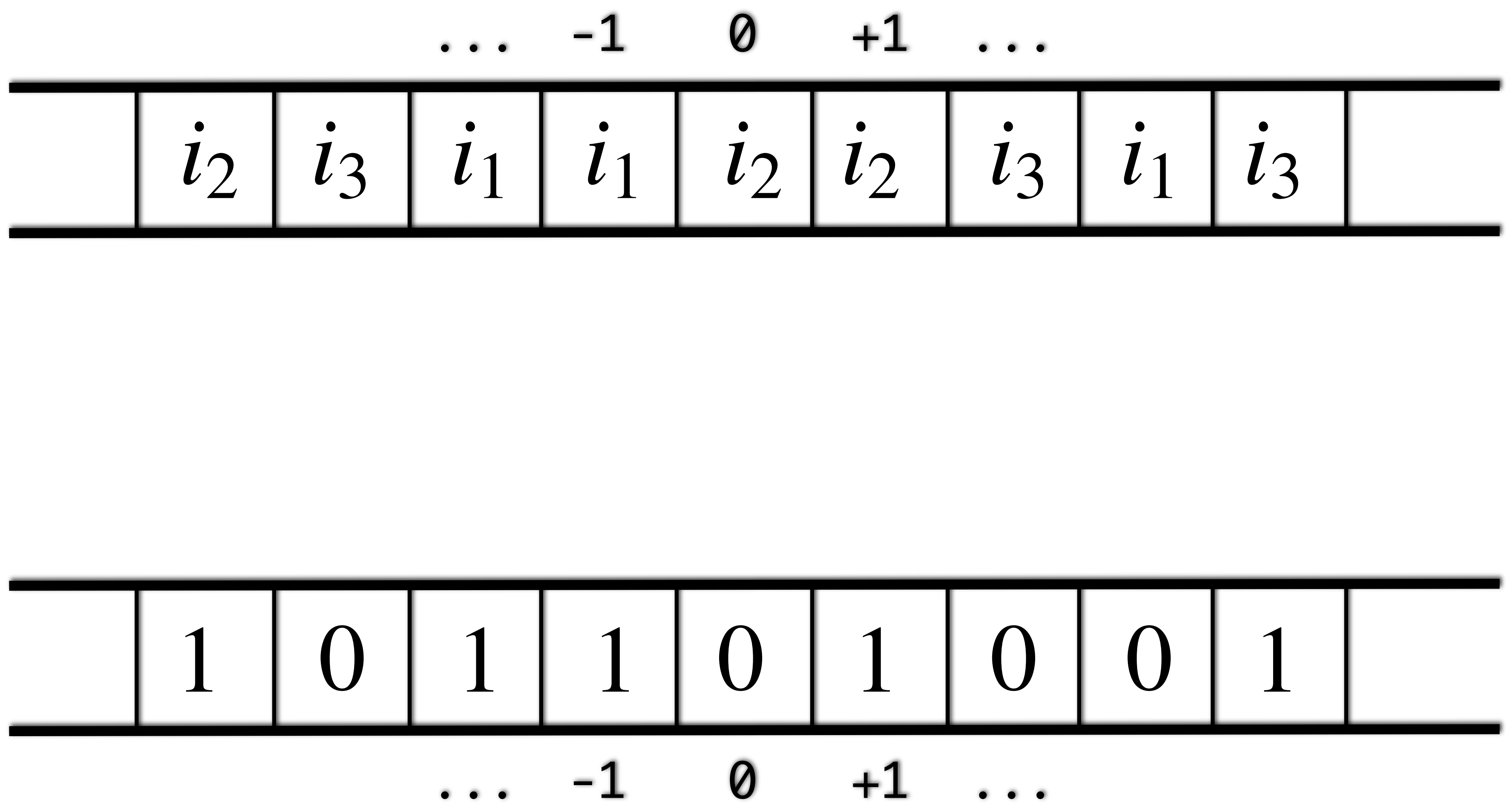


(Re-)Discoverings

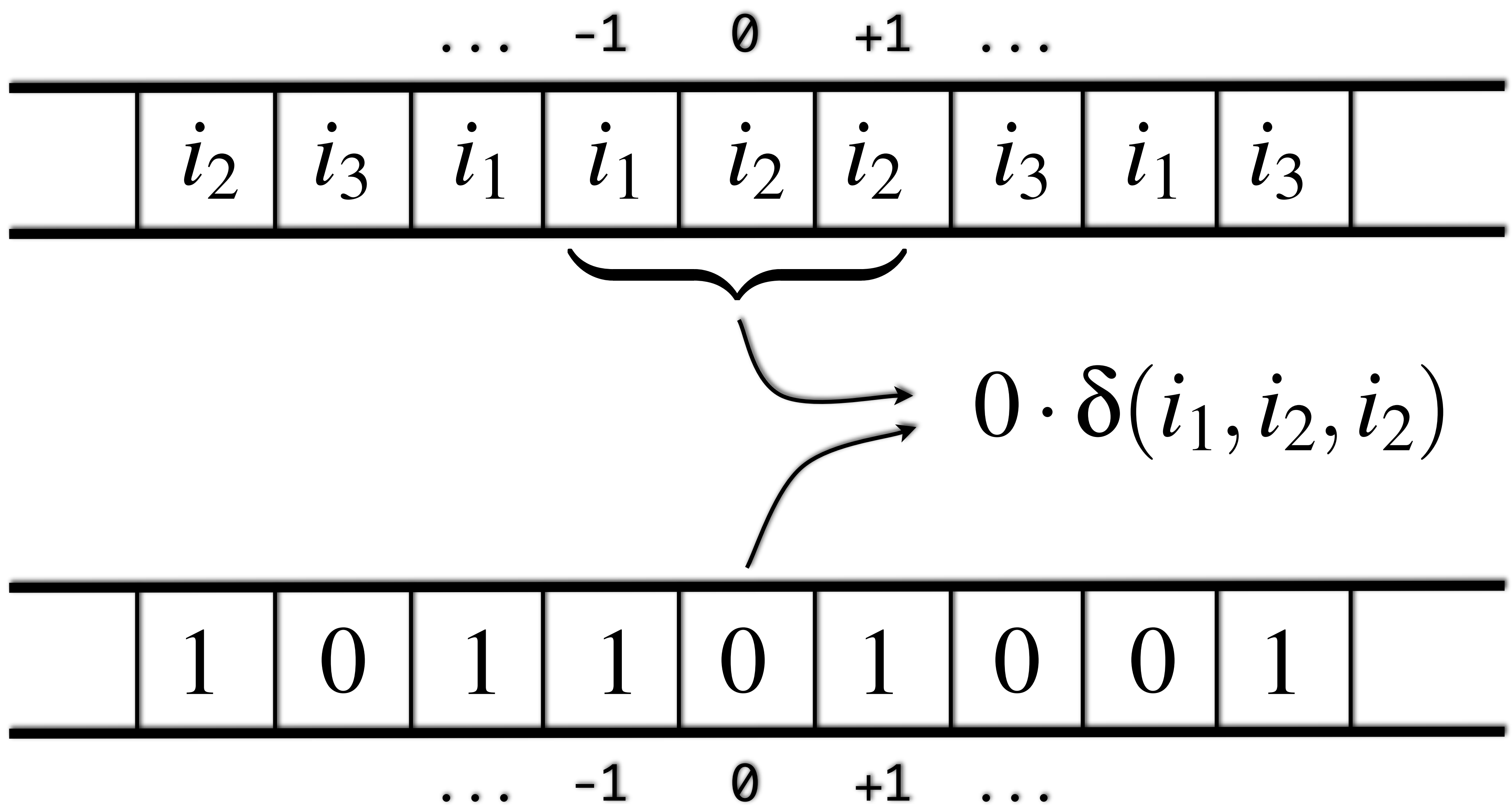
(Re-)Discoverings

			...	-1	0	+1	...			
	i_2	i_3	i_1	i_1	i_2	i_2	i_3	i_1	i_3	

(Re-)Discoverings



(Re-)Discoverings



More (re-)discoverings

$$F' \colon \{0,1\}^{\mathbb{Z}} \times I^{\mathbb{Z}} \to I^{\mathbb{Z}}$$

$$F'(x,y) = (id(x), F(y))$$

$$F'(x,y) = (G(x), F(y))$$

So far...

G shift invariant continuous ▸ CA

G continuous ▸ ν-CA

G shift invariant continuous ▸ “P”CA

G ▸ ???

So far...

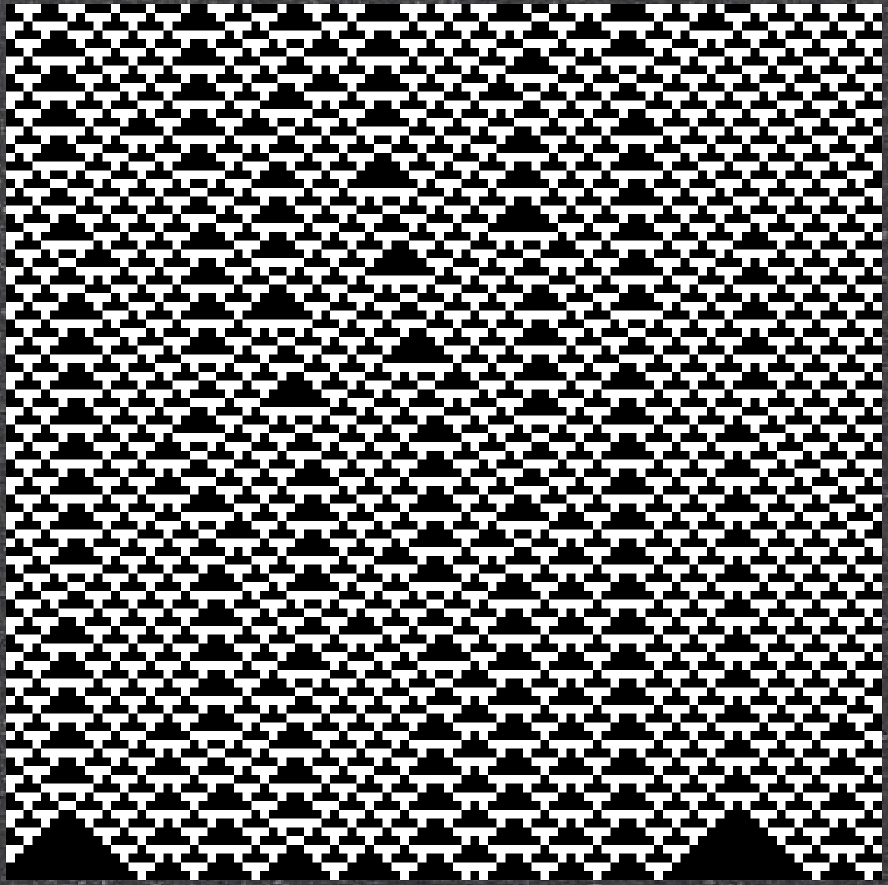
G shift invariant continuous ▸ CA

G continuous ▸ ν-CA

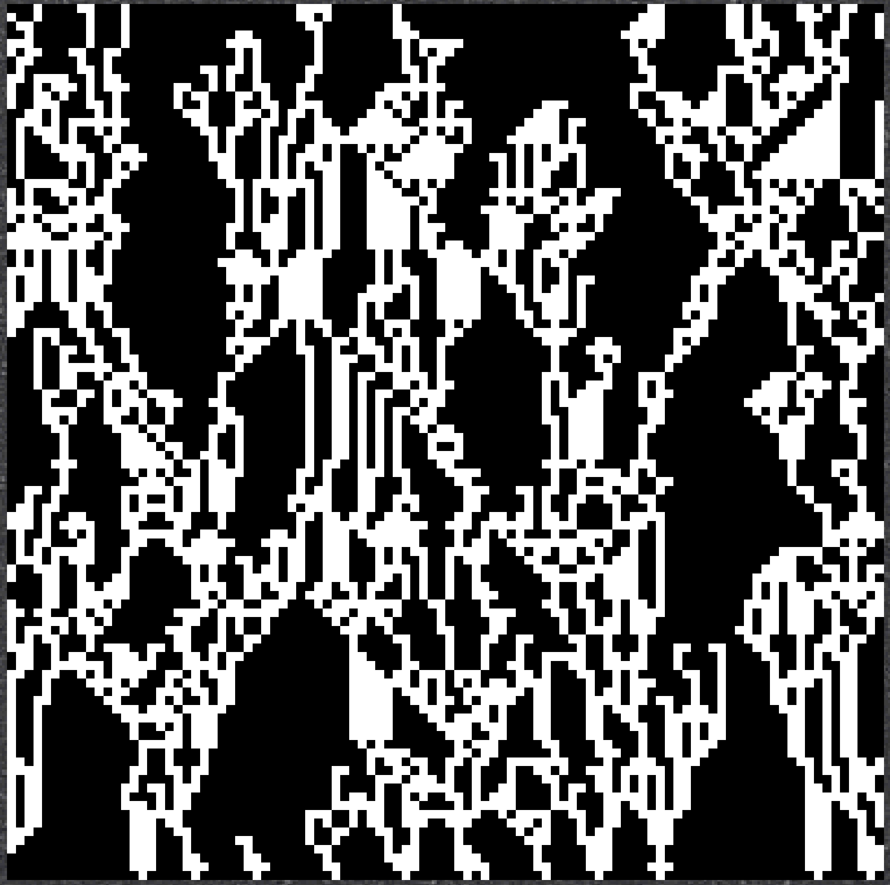
G shift invariant continuous ▸ "P"CA

G ▸ ???

Time for pictures

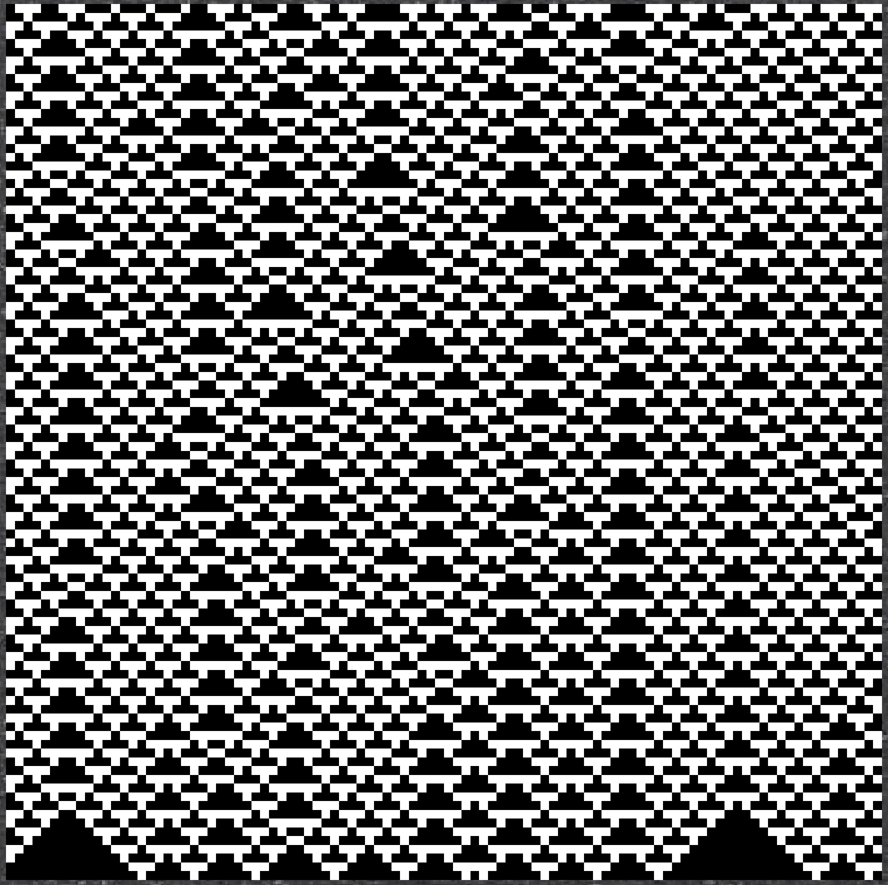


ECA 54

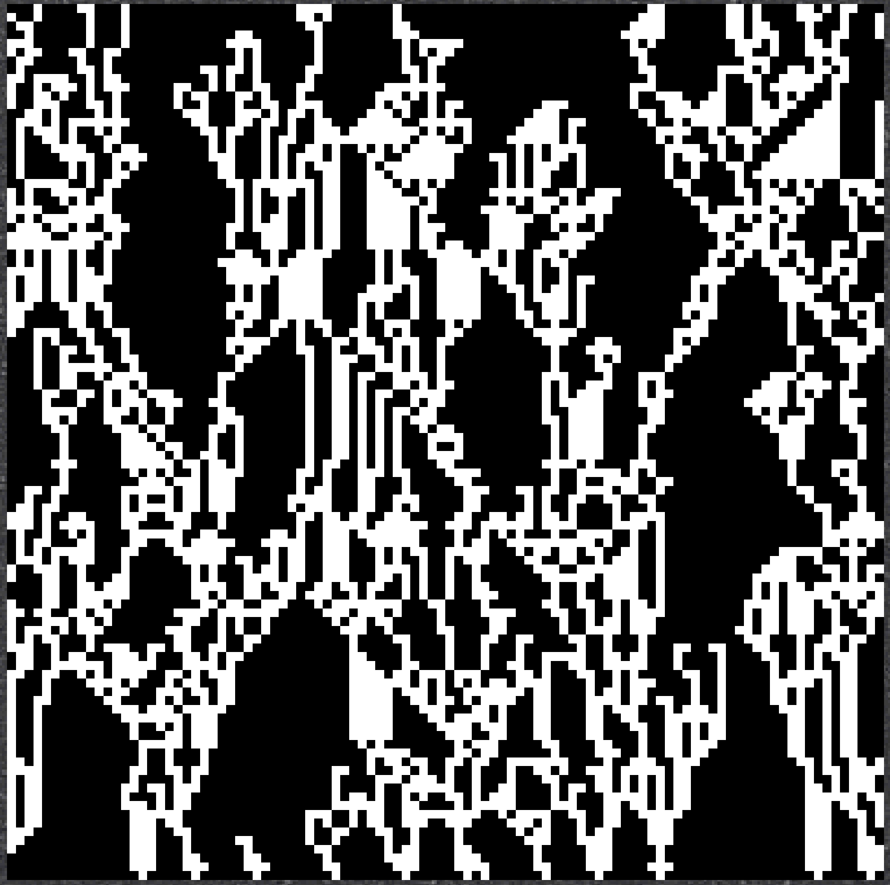


(54,90)

Time for pictures

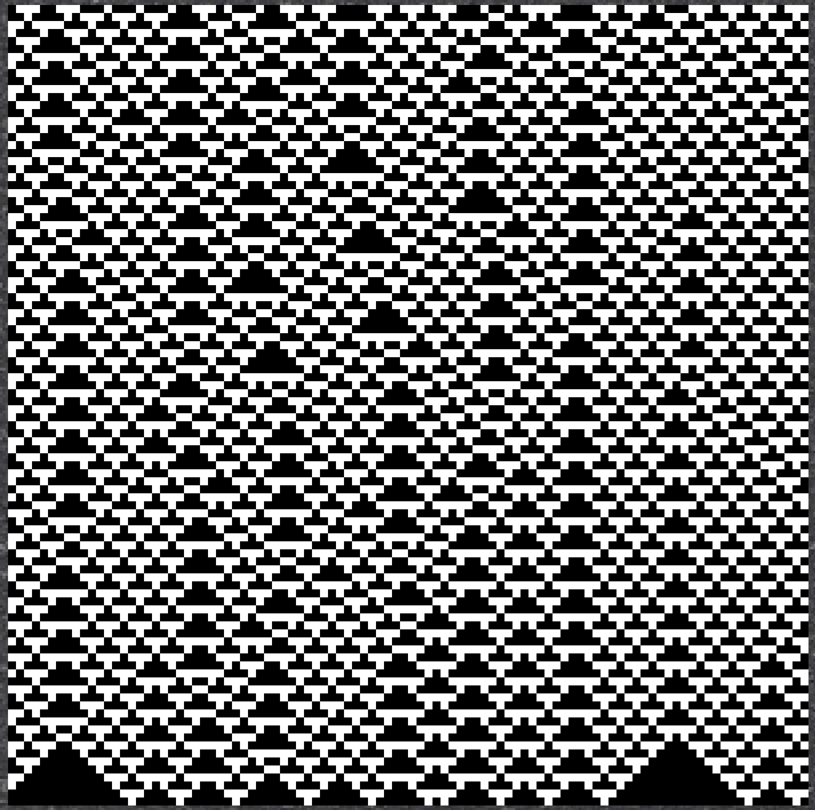


ECA 54

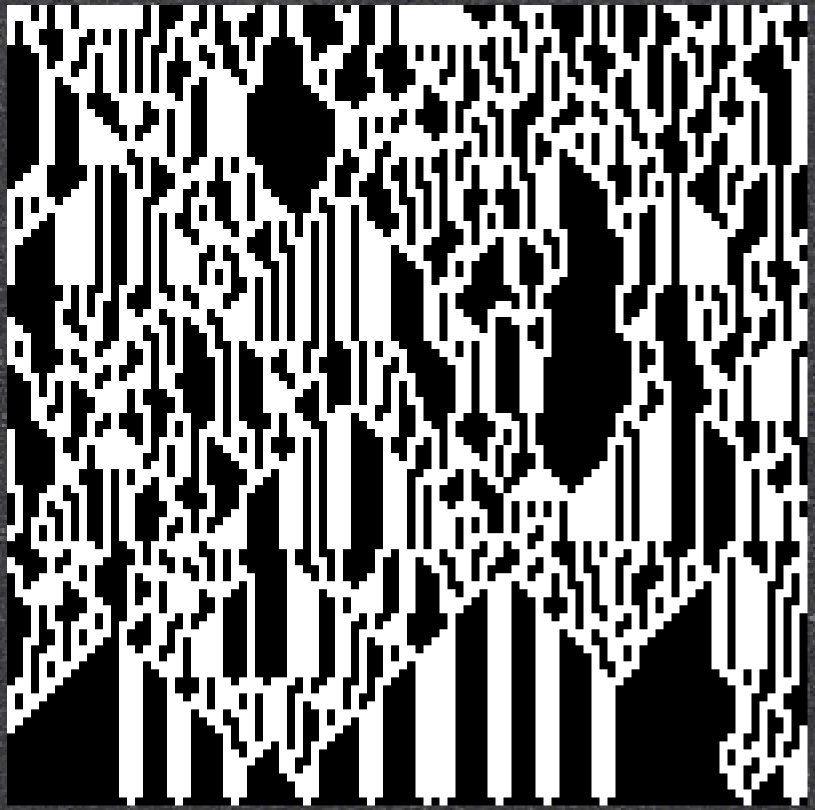


(54,90)

More pictures

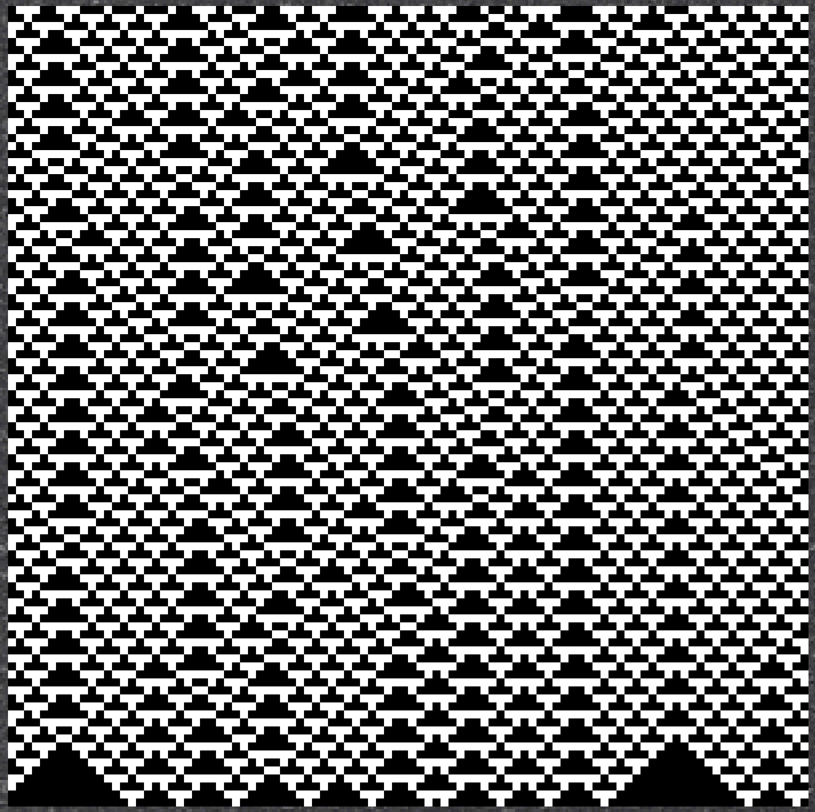


ECA 54

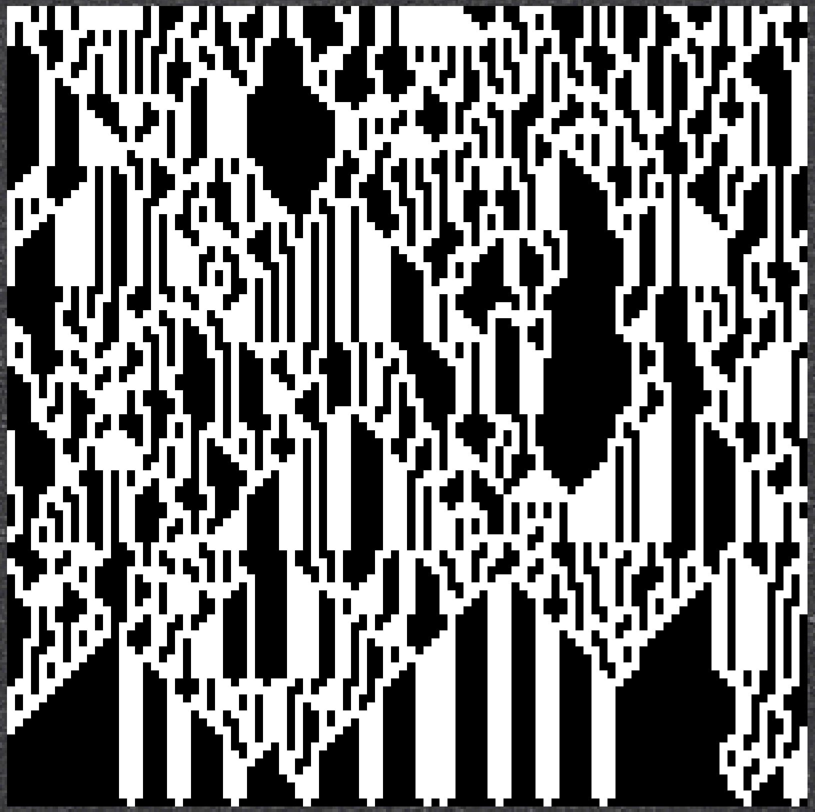


(54,18)

More pictures

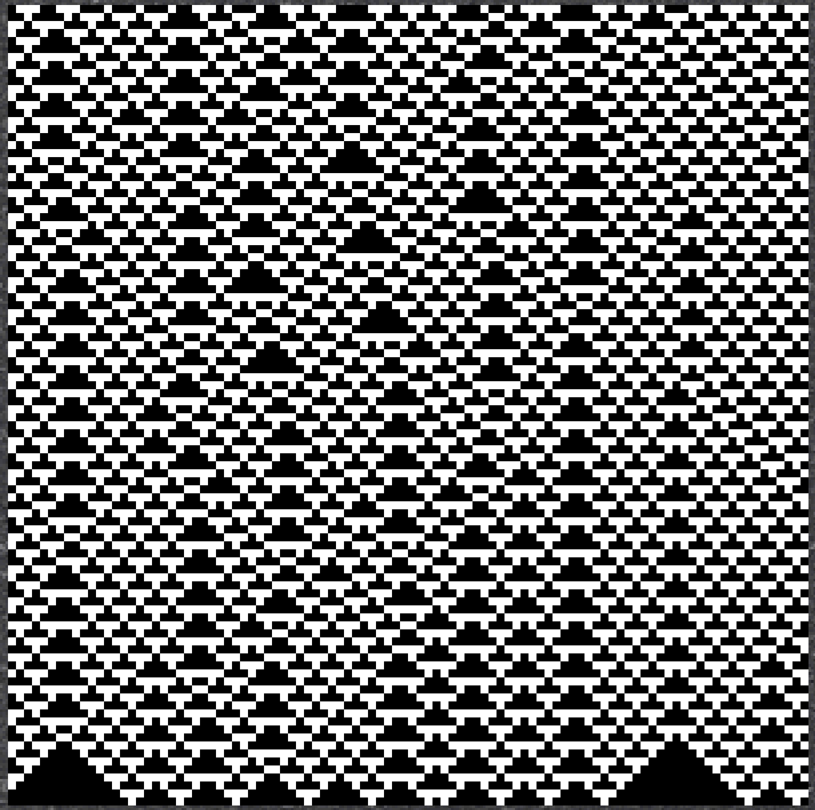


ECA 54

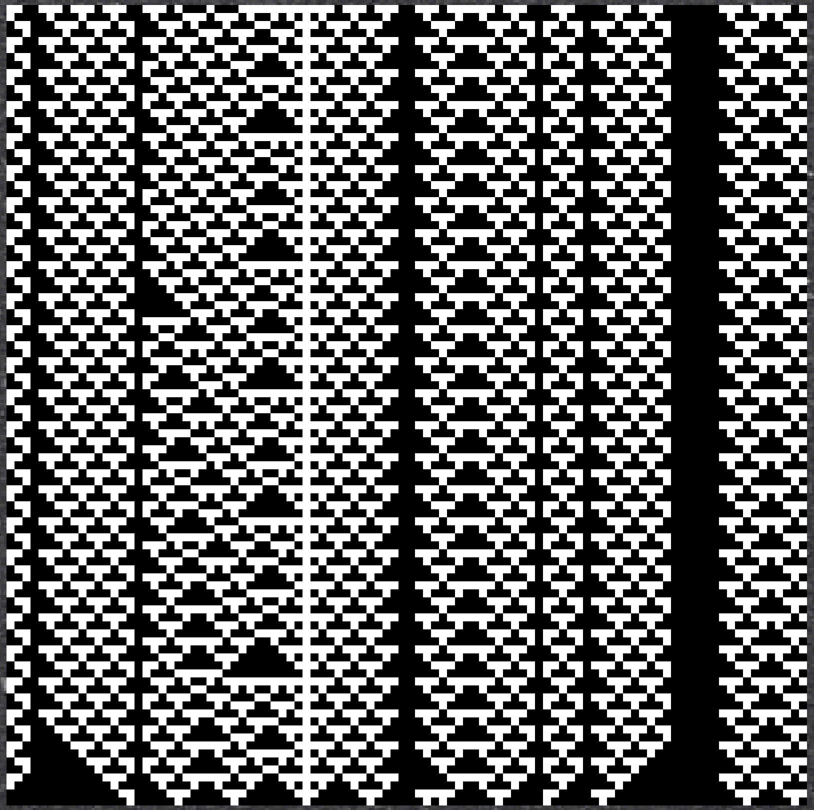


(54,18)

Even more pictures

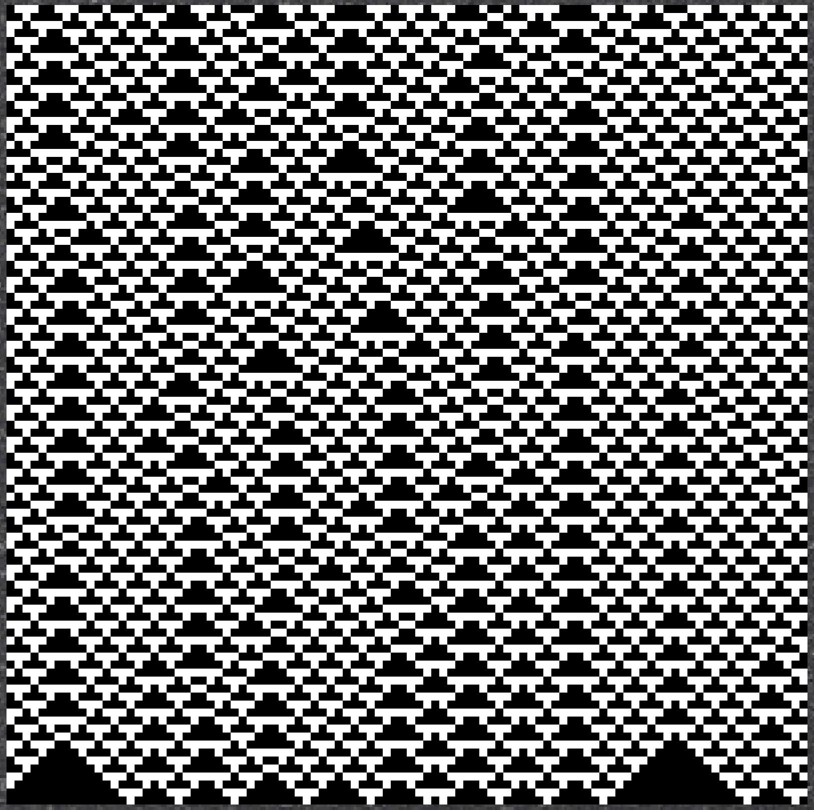


ECA 54

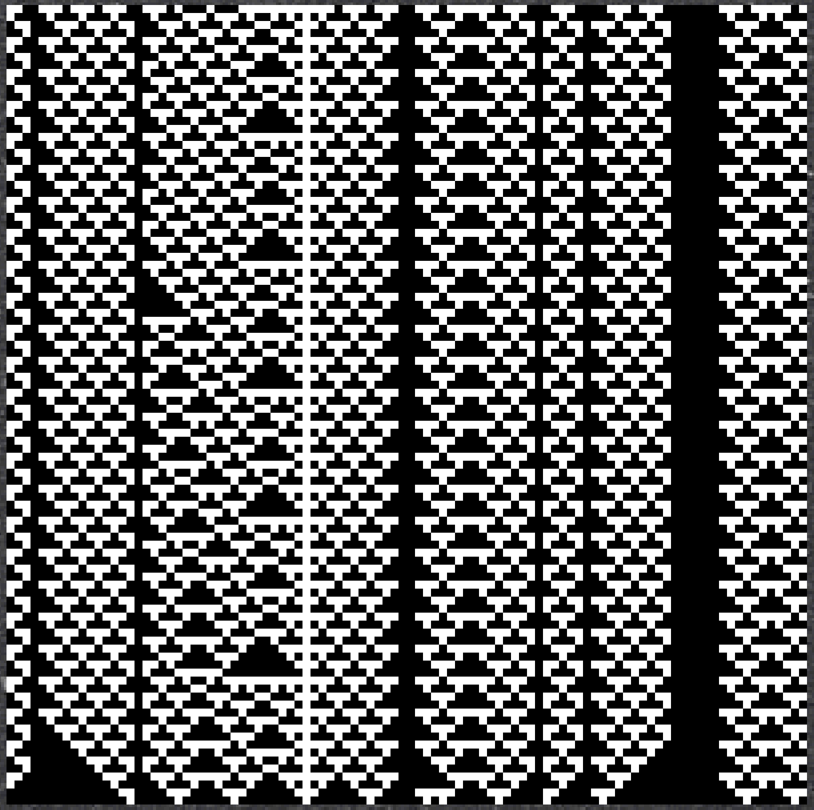


(54,id)

Even more pictures



ECA 54



(54,id)

Deterministic ACA

$$G = \mathbb{Z}^{\mathbb{N}}$$

$$t = 1 \quad \ldots 0,0,0,0,0,1,0|0,0,0,0,0,0\ldots$$

$$t = 0 \quad \ldots 0,0,0,0,0,0,0|0,0,0,0,1,0\ldots$$

$$t = 2 \quad \ldots 0,0,0,0,0,0,0|1,0,0,0,0,0\ldots$$

$$\vdots$$

Deterministic ACA

$G = \mathbb{Z}^{\mathbb{N}}$

$$t = 1 \qquad \ldots 0,0,0,0,0,1,0|0,0,0,0,0,0\ldots$$

$$t = 0 \qquad \ldots 0,0,0,0,0,0,0|0,0,0,0,1,0\ldots$$

$$t = 2 \qquad \ldots 0,0,0,0,0,0,0|1,0,0,0,0,0\ldots$$

$$\vdots$$

Global function

$$x_i^t$$

$$z_i^0 = y_i$$

$$z_i^t = (F')^t(x,y)_i$$

$$(F')^t(x,y)_i = (1-x_i^t)z_i^{t-1} + x_i^t\delta(z_{i-r}^{t-1},\dots,z_i^{t-1},\dots,z_{i+r}^{t-1})$$

Global function

$$x_i^t$$

$$z_i^0 = y_i$$

$$z_i^t = (F')^t(x,y)_i$$

$$(F')^t(x,y)_i = (1-x_i^t)z_i^{t-1} + x_i^t\delta(z_{i-r}^{t-1},\ldots,z_i^{t-1},\ldots,z_{i+r}^{t-1})$$

Set properties

Definition.

F' is injective iff

$$\forall y, z \in A^{\mathbb{Z}}, \forall x \in \mathbb{Z}^{\mathbb{N}}$$

$$z \neq y \Rightarrow \forall t \in \mathbb{N} \;\; F'(x^t, y) \neq F'(x^t, z)$$

Set properties

Definition.

F' is injective iff

$$\forall y, z \in A^{\mathbb{Z}}, \forall x \in \mathbb{Z}^{\mathbb{N}}$$

$$z \neq y \Rightarrow \forall t \in \mathbb{N} \;\; F'(x^t, y) \neq F'(x^t, z)$$

Set properties (2)

Definition.

F' is surjective iff

$\forall y \in A^{\mathbb{Z}}, \forall x \in \mathbb{Z}^{\mathbb{N}}$

$\forall t \in \mathbb{N} \; \exists z \in A^{\mathbb{Z}} \; F'(x^t, y) \neq F'(x^t, z)$

Set properties (2)

Definition.

F' is surjective iff

$\forall y \in A^{\mathbb{Z}}, \forall x \in \mathbb{Z}^{\mathbb{N}}$

$\forall t \in \mathbb{N}\ \exists z \in A^{\mathbb{Z}}\ F'(x^t, y) \neq F'(x^t, z)$

Set properties (3)

Proposition.

The following properties are equivalent

1) F' is injective

2) F' is surjective

3) δ is center permutative

Set properties (3)

Proposition.

The following properties are equivalent

1) F' is injective

2) F' is surjective

3) δ is center permutative

Dynamics

Proposition.

If $x \in \mathbb{Z}^{\mathbb{N}}$ is ultimately periodic, then

$$y, F'(x^1, y), (F')^2(x^2, y), \ldots, (F')^n(x^n, y), \ldots$$

is ultimately periodic.

Dynamics

Proposition.

If $x \in \mathbb{Z}^{\mathbb{N}}$ is ultimately periodic, then

$$y, F'(x^1, y), (F')^2(x^2, y), \ldots, (F')^n(x^n, y), \ldots$$

is ultimately periodic.

Dynamics (2)

Definition.

F' is sensitive to initial conditions, iff

$$\exists x \in \mathbb{Z}^{\mathbb{N}} \exists \varepsilon > 0 \forall y \in A^{\mathbb{Z}} \forall \delta > 0 \exists z \in \mathcal{B}_\delta(y) \exists t \in \mathbb{N}$$

such that

$$d((F')^t(x,y),(F')^t(x,z)) > \varepsilon$$

Dynamics (2)

Definition.

F' is sensitive to initial conditions, iff

$$\exists x \in \mathbb{Z}^{\mathbb{N}} \exists \varepsilon > 0 \forall y \in A^{\mathbb{Z}} \forall \delta > 0 \exists z \in \mathcal{B}_{\delta}(y) \exists t \in \mathbb{N}$$

such that

$$d((F')^t(x,y),(F')^t(x,z)) > \varepsilon$$

Dynamics (3)

Please look at the whiteboard on the right

Dynamics (3)

Please look at the whiteboard on the right

Dynamics (4)

Definition.

F' is expansive, iff

$$\exists x \in \mathbb{Z}^{\mathbb{N}} \exists \varepsilon > 0 \forall y \in A^{\mathbb{Z}} \forall z \in A^{\mathbb{Z}} \exists t \in \mathbb{N}$$

such that

$$d((F')^t(x,y),(F')^t(x,z)) > \varepsilon$$

Dynamics (4)

Definition.

F' is expansive, iff

$$\exists x \in \mathbb{Z}^{\mathbb{N}}\, \exists \varepsilon > 0 \forall y \in A^{\mathbb{Z}} \forall z \in A^{\mathbb{Z}} \exists t \in \mathbb{N}$$

such that

$$d((F')^t(x,y),(F')^t(x,z)) > \varepsilon$$

Dynamics (5)

Again
Please look at the whiteboard
on the right

Dynamics (5)

Again
Please look at the whiteboard
on the right

Dynamics (6)

Proposition.

Leftmost or rightmost permutative ACA are sensitive to initial conditions.

Proposition.

Leftmost and rightmost permutative ACA are expansive.

Dynamics (6)

Proposition.

Leftmost or rightmost permutative ACA are sensitive to initial conditions.

Proposition.

Leftmost and rightmost permutative ACA are expansive.

Dynamics (7)

Definition.

F' is transitive if and only if

$\exists x \in \mathbb{Z}^{\mathbb{N}} \quad \forall U, V \neq \emptyset \quad \exists t \in \mathbb{N}$ such that

$$(F')^t(x, U) \cap V \neq \emptyset$$

Dynamics (7)

Definition.

F' is transitive if and only if

$\exists x \in \mathbb{Z}^{\mathbb{N}} \quad \forall U, V \neq \emptyset \quad \exists t \in \mathbb{N}$ such that

$$(F')^t(x, U) \cap V \neq \emptyset$$

Dynamics (8)

Look at the whiteboard

Dynamics (8)

Look at the whiteboard

Dynamics (9)

Proposition.

Leftmost or rightmost permutative ACA are transitive.

(Recall that they are also sensitive)

Dynamics (9)

Proposition.

Leftmost or rightmost permutative ACA are transitive.

(Recall that they are also sensitive)

Dynamics (10)

Definition.

F' has the DPO property if and only if $\exists x \in \mathbb{Z}^{\mathbb{N}}$ such that $F'(x, \cdot)$ has the DPO property.

$F'(x, \cdot)$ has the DPO property if and only if its set of periodic points is dense.

Dynamics (10)

Definition.

F' has the DPO property if and only if $\exists x \in \mathbb{Z}^{\mathbb{N}}$ such that $F'(x,\cdot)$ has the DPO property.

$F'(x,\cdot)$ has the DPO property if and only if its set of periodic points is dense.

Dynamics (11)

Proposition.

Deterministic ACA have DPO iff they are surjective.

Dynamics (11)

Proposition.

Deterministic ACA have DPO iff they are surjective.

Dynamics (12)

Lemma.

If $F' \neq id$ and has DPO for some $x \in \mathbb{Z}^{\mathbb{N}}$ then x is bounded.

Corollary.

Fix $x \in \mathbb{Z}^{\mathbb{N}}$. Then $F'(x, \cdot)$ cannot be Devaney chaotic.

Dynamics (12)

Lemma.

If $F' \neq id$ and has DPO for some $x \in \mathbb{Z}^{\mathbb{N}}$ then x is bounded.

Corollary.

Fix $x \in \mathbb{Z}^{\mathbb{N}}$. Then $F'(x, \cdot)$ cannot be Devaney chaotic.

Beginning to conclude

Deterministic ACA are interesting!

What about

Beginning to conclude

Deterministic ACA are interesting!

What about

- Nilpotency ?

Beginning to conclude

Deterministic ACA are interesting!

What about

- Nilpotency ?
- Topological entropy ?

Beginning to conclude

Deterministic ACA are interesting!

What about

- Nilpotency ?
- Topological entropy ?
- Classification ?

Beginning to conclude

Deterministic ACA are interesting!

What about

- Nilpotency ?
- Topological entropy ?
- Classification ?
- Higher dimensions ?

Continuing to conclude

Updating schemes

Continuing to conclude

Updating schemes

- More fairness

Continuing to conclude

Updating schemes

- More fairness
- Structural properties

Continuing to conclude

Updating schemes

- More fairness
- Structural properties
- Applications (?)

True conclusions

About computability

- Decidability
- Tradeoffs

The End.

Many thanks for your attention!